The Sedimentation-Dispersion Model for Slurry Bubble Columns

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Slurry bubble columns have been widely used in petrochemical and coal conversion industries, and increasingly applied in fermentation and wastewater treatment fields. A slurry bubble column operated under the expanded bed regime (Fan et al., 1987) is characterized by an axial solids concentration distribution in the column. Quantitative description of this distribution has been based mainly on the sedimentation-dispersion (*S-D*) model, originally proposed by Cova (1966) and Suganuma and Yamanishi (1966). In the model, the solids flux is obtained by superposition of a Fickian type solids dispersion flux and a solids settling (sedimentation) flux on the convective slurry flux. The model is, in essence, phenomenological, and various formulations have resulted in discrepancies in the interpretation of the parameters in the model, e.g., the solids settling velocity. The solids settling velocity parameter has been interpreted either as the particle terminal velocity (e.g., Cova, 1966), or as the hindered settling velocity of a particle swarm relative to the liquid (e.g., Brian and Dyer, 1984). Parulekar and Shah (1980) defined the solids settling velocity parameter as the hindered solids settling velocity relative to the column, whereas Smith and Ruether (1985) defined it as the solids velocity relative to either the slurry or the liquid velocity.

In this note, the physical nature of the *S-D* model in the literature is discussed in light of a rigorous analysis of the axial solids concentration distribution based on a macroscopic balance of the solid phase. General validity of the model for various modes of operation of slurry bubble column systems is also discussed.

Formulation Based on Macroscopic Solids Balance

Consider three modes of operation for the slurry bubble column, namely, batch mode with respect to solids and liquids (denoted as Mode *B*-I), batch mode with respect to solids alone (denoted as Mode *B*-II), continuous flow mode with respect to solids and liquids (denoted as Mode *C*). The axial mixing of solids in a multiphase system can be expressed by directly extending the approach proposed by Cussler (1984), which describes the axial dispersion of solutes in turbulent flow systems. For a slurry bubble column with polydispersed particles in each mode of operation, it is assumed that the solid phase can be treated as a continuum. A continuous distribution of particle size can be grouped into discrete, narrow fractions of the particle size with each fraction denoted generally by i. The continuity equation for the flow of the ith fraction of solid particles may be written in terms of solids holdup as

$$\frac{\partial \tilde{\epsilon}_{si}}{\partial t} = -\nabla \cdot (\underline{\tilde{V}}_{pi}\tilde{\epsilon}_{si}) \tag{1}$$

Both $\underline{\tilde{V}}_{pi}$ and $\tilde{\epsilon}_{si}$ fluctuate with time and can be represented as the sum of their average values and fluctuating components

$$\underline{\tilde{V}}_{pi} = \underline{V}^*_{pi} + \underline{v}'_{pi} \quad \text{and} \quad \tilde{\epsilon}_{si} = \epsilon_{si} + \epsilon'_{si} \tag{2}$$

Substituting Eq. 2 into Eq. 1 and applying averaging rules results in

$$\frac{\partial \epsilon_{si}}{\partial t} = -\nabla \cdot (\underline{V}^*_{pi}\epsilon_{si}) - \nabla \cdot (\overline{\underline{v}'_{pi}\epsilon'_{si}}) \tag{3}$$

where $(\overline{\underline{v}'_{pi}\epsilon'_{si}})$ is the time average of the fluctuation product. The flux $(\overline{\underline{v}'_{pi}\epsilon'_{si}})$ bears a similar physical significance to the diffusional flux vector, and can thus be defined as

$$\overline{\underline{v}'_{pi}\epsilon'_{si}} = -E_{rsi}\frac{\partial \epsilon_{si}}{\partial r}\hat{r} - \frac{E_{\theta si}}{r}\frac{\partial \epsilon_{si}}{\partial \theta}\hat{\theta} - E_{zsi}\frac{\partial \epsilon_{si}}{\partial z}\hat{z} \tag{4}$$

Combining Eqs. 3 and 4, performing the surface integration over the cross sectional area of the column, and dividing each

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term by the column cross sectional area yields:

$$\frac{\partial \epsilon_{si}}{\partial t} = -\frac{\partial}{\partial z}\left(\frac{\iint V^*_{pi}\epsilon_{si} r dr d\theta}{A}\right) + \frac{\partial}{\partial z}\left(E_{zsi}\frac{\partial \epsilon_{si}}{\partial z}\right) \tag{5}$$

Assuming constant ϵ_{si} with respect to r and θ gives:

$$\frac{\partial \epsilon_{si}}{\partial t} = \frac{\partial}{\partial z}\left[E_{zsi}\frac{\partial \epsilon_{si}}{\partial z} - V_{pi}\epsilon_{si}\right] \tag{6}$$

Note that V_{pi} is a cross-sectional average of the z component of the particle velocity vector, and E_{zsi} is defined based on the cross-sectional area of the column. ϵ_{si} is related to the solids concentration in the slurry, C_{si}, by

$$C_{si} = \{\epsilon_{si}/(1 - \epsilon_g)\}\rho_{si} \tag{7}$$

V_{pi} can be related to the relative velocity between the slurry and the ith fraction of solid particles, U'_{ti} or U''_{ti}, by

$$V_{pi} = \frac{\overline{U}^*_{sl}}{1 - \epsilon_g} - U'_{ti} \quad \text{or} \quad V_{pi} = \frac{\overline{U}_{sl}}{1 - \epsilon_g} - U''_{ti} \tag{8}$$

Two types of average slurry velocity exist, namely the weight-average superficial slurry velocity, $\overline{U}_{sl}$, and the volume-average superficial slurry velocity, $\overline{U}^*_{sl}$. They are given as follows:

$$\frac{\overline{U}_{sl}}{1 - \epsilon_g} = \frac{\sum_{j=1}^{n} V_{pj}\epsilon_{sj}\rho_{sj} + V_l\epsilon_l\rho_l}{\sum_{j=1}^{n} \epsilon_{sj}\rho_{sj} + \epsilon_l\rho_l}$$

$$\text{and} \quad \frac{\overline{U}^*_{sl}}{1 - \epsilon_g} = \frac{\sum_{j=1}^{n} V_{pj}\epsilon_{sj} + V_l\epsilon_l}{1 - \epsilon_g} \tag{9}$$

Note that $\overline{U}^*_{sl}$ is readily measurable compared to $\overline{U}_{sl}$, and is thus more commonly used. V_{pi} in Eq. 6 can also be related to the average slurry velocity and the slip velocity between liquid and each solid fraction via the mass conservation relationship. The relative (or slip) velocity between the liquid and solid particles of the ith fraction, U_{pi}, is given by

$$U_{pi} = V_l - V_{pi} \quad i = 1, 2, \ldots, n \tag{10}$$

Combining Eqs. 9 and 10, the average solid convective velocity of the ith particle fraction can be related to the average superficial slurry velocities and the solid-liquid relative velocity by

$$V_{pi} = \frac{\overline{U}^*_{sl}}{(1 - \epsilon_g)} - U_{pi} + \sum_{j=1}^{n} U_{pj}\frac{\epsilon_{sj}}{(1 - \epsilon_g)}$$

$$\text{or} \quad V_{pi} = \frac{\overline{U}_{sl}}{(1 - \epsilon_g)} - U_{pi} + \frac{\sum_{j=1}^{n} U_{pj}\epsilon_{sj}\rho_{sj}}{\sum_{j=1}^{n} \epsilon_{sj}\rho_{sj} + \epsilon_l\rho_l} \tag{11}$$

Considering a constant axial gas holdup, which is evident from the data of Smith and Ruether (1985), and combining Eqs. 6, 7, and 11, yield

$$\frac{\partial C_{si}}{\partial t} = \frac{\partial}{\partial z}\left\{E_{zsi}\frac{\partial C_{si}}{\partial z} - \left[\frac{\overline{U}^*_{sl}}{(1 - \epsilon_g)} - U_{pi} + \sum_{j=1}^{n}\psi_{sj}U_{pj}\right]C_{si}\right\} \tag{12a}$$

or

$$\frac{\partial C_{si}}{\partial t} = \frac{\partial}{\partial z}\left\{E_{zsi}\frac{\partial C_{si}}{\partial z} - \left[\frac{\overline{U}_{sl}}{(1 - \epsilon_g)} - U_{pi} + \sum_{j=1}^{n}\omega_{sj}U_{pj}\right]C_{si}\right\} \tag{12b}$$

$$\text{where} \quad \psi_{sj} = \frac{\epsilon_{sj}}{1 - \epsilon_g} \quad \text{and} \quad \omega_{sj} = \frac{\epsilon_{sj}\rho_{sj}}{\sum_{j=1}^{n}\epsilon_{sj}\rho_{sj} + \epsilon_l\rho_l} \tag{13}$$

For a monodispersed system, Eqs. 12a and 12b become, respectively,

$$\frac{\partial C_s}{\partial t} = \frac{\partial}{\partial z}\left\{E_{zs}\frac{\partial C_s}{\partial z} - \left[\frac{\overline{U}^*_{sl}}{(1 - \epsilon_g)} - \psi_l U_p\right]C_s\right\} \tag{14a}$$

$$\frac{\partial C_s}{\partial t} = \frac{\partial}{\partial z}\left\{E_{zs}\frac{\partial C_s}{\partial z} - \left[\frac{\overline{U}_{sl}}{(1 - \epsilon_g)} - \omega_l U_p\right]C_s\right\} \tag{14b}$$

$$\text{where} \quad \psi_l = \frac{\epsilon_l}{1 - \epsilon_g} \quad \text{and} \quad \omega_l = \frac{\epsilon_l\rho_l}{\epsilon_s\rho_s + \epsilon_l\rho_l} \tag{15}$$

and V_p for a monodispersed system becomes

$$V_p = V_l - U_p = \frac{\overline{U}^*_{sl}}{1 - \epsilon_g} - \psi_l U_p \tag{16}$$

For Mode B-I ($\overline{U}^*_{sl} = 0$) at a steady-state condition, Eq. 14a reduces to

$$\frac{\partial}{\partial z}\left\{E_{zs}\frac{\partial C_s}{\partial z} + \psi_l U_p C_s\right\} = 0 \tag{17}$$

For Mode B-II at a steady-state condition, Eq. 14a becomes

$$\frac{\partial}{\partial z}\left\{E_{zs}\frac{\partial C_s}{\partial z} - \psi_l[V_l - U_p]C_s\right\} = 0 \tag{18}$$

Discussion

Table 1 summarizes various forms of the S-D model reported in the literature. As can be seen in the table, the S-D model is characterized in the literature by two parameters, denoted generally here as E'_{zs} and U'_t. Note that V_l and $U_{t\infty}$ terms appearing in Table 1 can be regarded as equivalent forms of $\overline{U}^*_{sl}/(1 - \epsilon_g)$ and U'_t, respectively, for low solids concentration conditions. Substituting Eqs. 7 and 8 into Eq. 6 in the present analysis under constant axial gas holdup and monocomponent conditions yields an expression which is essentially of the same parametric form as that given in the S-D model, with E_{zs} and U'_{tm} (corresponding to U'_{ti} for a monocomponent system) corresponding to E'_{zs} and U'_t, respectively. Though based on the same phenomenological starting point indicated in the opening of this note, other, less rigorous, derivations of the S-D model have resulted in inconsistent definition and interpretation of the model parameters. More specifically, the parameter U'_t in the S-D model has been

Table 1. Various Forms of the Sedimentation-Dispersion Model

General Equation: $\frac{\partial C_s}{\partial t} = \frac{\partial}{\partial z}\left\{E \frac{\partial C_s}{\partial z} - [V - U] C_s\right\}$

Authors	E	V	U	Comments
	Parameters			
Cova (1966)	E'_{zs}	V_l	$U_{t\infty}$	—
Suganuma & Yamanishi (1966)	E'_{zs}	V_l	U'_t	—
Imafuku et al. (1968)	E'_{zs}	V_l	U'_t	—
Farkas and Leblond (1969)	E'_{zs}	0	U'_t	For Mode B-I
Kato et al. (1972, 1985)	E'_{zs}	$\frac{\overline{U}^*_{sl}}{1 - \epsilon_g}$	U'_t	—
Kafarov et al. (1973)	E'_{zs}	V_l	U'_t	For Mode B-II
Parulekar & Shah (1980)	E''_{zs}	$\frac{\overline{U}_{sl}}{1 - \epsilon_g}$	U'_{st}	Mispairing of $\overline{U}_{sl}$ with U'_{st} ($\psi_l U_p$) (cf., Eq. 14b).
Kojima et al. (1984)	E'_{zs}	V_l	$U_{t\infty}$	—
Brian & Dyer (1984)	E'_{zs}	V_l	U'_t	—
Smith & coworkers (1985, 1986†)	E''_{zs}	$\frac{\overline{U}^*_{sl}}{1 - \epsilon_g}$	U'_t	$U'_t = \psi_l U_p$
	E''_{zsi}	$\frac{\overline{U}^*_{sl}}{1 - \epsilon_g}$	$\psi'_l U_{pi}$	†For a polydispersed system
Ueyama et al. (1985)	E'_{zs}	V_l	U'_t	For Mode B-II

subject to various differing interpretations, and hence expressions, resulting in substantial confusion regarding application of the S-D model. It is important to note that the numerical values of the S-D model parameters are obtained simultaneously by fitting the model directly to experimental data of the axial solids concentration. The resulting numerical values for model parameters in the S-D model are in many cases not physically reasonable or compatible in terms of definition and magnitude, specifically, for Mode B-I, Mode B-II and some instances of Mode C. These values are given in light of present rigorous formulations.

In the literature, most of the experimental axial solids concentration profiles, $C_s(z)$, have been reported for slurry-batch, solids-batch, and slow slurry continuous flow conditions, which usually operate in the coalesced bubble flow regime. The axial solids profile is characterized by a concave upward curve. This profile gives rise to a negative first derivative of C_s with respect to z, and a positive second derivative. To fit the solids concentration profile for Mode C operation based on the steady state form of Eq. 14a, V_p, defined by Eq. 16 must be negative, indicating a downward solids convective velocity. The resulting negative V_p is in conflict with the positive V_p expressed in the original formulation for a continuous slurry flow.

For Modes B-I and B-II operation, described by Eqs. 17 and 18 respectively, V_p is equal to $-\psi_l U_p$ (Mode B-I) or $\psi_l(V_l - U_p)$ (Mode B-II). Note that for Modes B-I and B-II, there is no net flow of solids through the system. The convective solids velocity, V_p, must be equal to zero. Consequently, the convective terms in both Eqs. 17 and 18 are equal to zero, resulting in an axially uniform solids concentration. The model therefore breaks down and can not be used to describe the solids concentration profiles for batch operation. The corresponding V_p in the S-D model in the literature, i.e., U'_t (or $V_l - U'_t$), however, was retained intact and forced to nonzero values in data fitting. U'_t was further interpreted indistinctly as the solids sedimentation velocity (e.g., Kato et al., 1985; and Ueyama et al., 1985).

Based on the above discussion and the large amount of supporting experimental data on the axial solids concentration distribution, it can be stated that under the coalesced bubble flow regime and slow slurry flow conditions, the parameters in the S-D model, E'_{zs} and U'_t, should be regarded as purely empirical and as bearing no physical significance. Recognizing this fact, the correlations reported in the literature to estimate these model parameters still present a useful means for predicting the axial slurry concentration profile in the systems. Note that, based on the present formulation, physically compatible model parameters can only be obtained under the fast slurry flow and dispersed bubble flow regime conditions, though available experimental data are insufficient to confirm this.

In contrast to the phenomenological approach in the S-D model development, several attempts have been made to develop mechanistically based models for slurry bubble columns (e.g., El-Temtamy and Epstein, 1980; Tang and Fan, 1987; Murray and Fan, 1988). For Mode B-II operation, Tang and Fan (1987) considered that the three-phase fluidized bed with low particle density consists of three distinct phases: gas, bubble-engagement, and the particulate fluidization phases. Based on the solids mass balance of the bubble-engagement and particulate fluidization phases, they arrived at a model equation similar to Eq. 18, with E_{zs} accounting for the solid upward dispersion resulting from the solid entrainment and deentrainment associated with bubbles, and V_p (or $V_l - U_p$ in Eq. 18) representing the solid settling in the particulate fluidization region. Their model was also shown to be extensible to the coalesced bubble or part of the dispersed bubble regimes in Mode C of the slurry bubble columns (Murray and Fan, 1988).

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Notation

A = cross-sectional area of column

C_s = solids concentration in slurry for a monodispersed system; mass of solids per unit volume of slurry

E_{rsi} = solid-phase radial dispersion coefficient of the ith particle fraction for polydispersed slurry system based on the column's cross-sectional area
$E_{\theta si}$ = as above, for tangential dispersion coefficient
E_{zs} = solid-phase axial dispersion coefficient for monodispersed slurry system based on the column's cross-sectional area
E'_{zs} = solid-phase axial dispersion coefficient
E''_{zs} = solid-phase axial dispersion coefficient for monodispersed slurry system based on the slurry's cross-sectional area
i = for the ith particle fraction in a polydispersed system
n = number of fractions in polydispersed system
r = radial component of cylindrical coordinates
$\hat{r}$ = unit vector in r-direction
t = time
U_p = average velocity: solids over the column's cross-sectional area vs. liquid for a monodispersed system
$\bar{U}^*_{sl}$ = volume-average superficial slurry velocity based on the column's cross-sectional area for mono- and polydispersed systems
$\bar{U}_{sl}$ = weight-average superficial slurry velocity
U'_t = solids-settling velocity
U'_{st} = settling velocity relative to column, Parulekar and Shah (1980)
U'_{ti} = volume-average linear slurry velocity vs. average solids convective velocity for the ith particle fraction in a polydispersed system
U'_{tm} = as above, for a monocomponent system
U''_{ti} = as above, with weight average slurry velocity
$U_{t\infty}$ = terminal settling velocity of a particle in an infinite extent of medium
V_l = cross-sectional average of linear liquid velocity
V_p = cross-sectional average of z component of time-averaged convective velocity vector of solid particles relative to column for monodispersed system, positive if upward
$\underline{V}^*_{pi}$ = time-averaged solids convective velocity vector relative to column for the ith particle fraction in polydispersed system
$V^*_{pi}, V^*_{ri}, V^*_{\theta i}$ = z, r and θ components of $\underline{V}^*_{pi}$
$\tilde{\underline{V}}_{pi}$ = instantaneous solids convective velocity vector relative to column for the ith particle fraction in polydispersed system
$\underline{v}'_{pi}$ = fluctuation velocity vector relative to the mean ($\underline{V}^*_{pi}$)
z = axial coordinate in cylindrical system, positive if upward
$\hat{z}$ = unit vector in z-direction

Greek letters

ϵ = subscripts g, l for gas, and liquid holdups
ϵ_s = time-averaged axial solids holdup for monodispersed system
$\tilde{\epsilon}_{si}$ = instantaneous axial solids holdup of the ith particle fraction in polydispersed system
ϵ'_{si} = fluctuation of solids holdup from time-averaged value
ρ_l = liquid density
ρ_s = solid density for monodispersed system
ψ_l = liquid volume fraction in slurry for a monodispersed system
ψ'_l = as above, for polydispersed system
ω_l = liquid weight fraction in slurry for monodispersed system
ω_{si} = weight-average solids fraction in slurry for the ith fraction in polydispersed system
ψ_{si} = volume-average solids fraction
θ = θ component of cylindrical coordinates
$\tilde{\theta}$ = unit vector in θ-direction

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